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言語と脳科学への位相数学的アプローチ

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Topological Approach to the Science of Language and Brain

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脳科学の発展に伴い、多くの研究者は人間の精神活動を脳の神経細胞（neuron）あるいはその集合体（cell assembly）や神経回路網（neural net）の活動という微視的な観点から説明しようと試みてきた。他方で、非侵襲的なニューロイメジングの技術開発により生きた脳の活動（血流量や代謝量の変化）を巨視的に観察し、それぞれの精神活動（認知過程）に対応する脳領域を特定することが可能となり、新たなモデルの展開に拍車をかけた。しかし、人間の精神現象（意識）がすなわちニューロン群の活動であるといった一元論的な見方をしない限り、内的な意識現象そのものを客観的に説明することは不可能と思われる。そこで、内的意識過程を表現可能にし、したがって外部から客観的に観察しうる言語を研究対象とし、その言語活動の分析に位相数学を応用することで人間の高次精神活動を科学的に研究できるのではないかと考え、それを一つの試論として提示した。

Key Words： Topology, Language, Brain, Meaning, Set, Mapping
トポロジー、言語、脳、意味、集合、写像

Since the end of the 20th century the development of brain imaging techniques, such as PET (Positron Emission Tomography), MEG (Magneto-encephalography), and fMRI (Functional Magnetic Resonance Imaging)[1), has contributed to the rapid progress of the scientific research on neural and mental processes in the living brain[2). Consequently, the brain science of today can bring about a better understanding of the linkage between brain and mind, in cooperation with various scientific fields such as psychology, neurophysiology, cognitive neuroscience, brain pathology, psychiatry, molecular biology, biochemistry, quantum dynamic theory, and so on. Naturally, almost all the scientists in those fields seem to hold their monistic expectation that the brain mechanisms generating conscious phenomena will be physically and objectively explained in the near future.

As the brain science develops, however, the problem of ‘ consciousness ’ comes to reappear among researchers engaging in the investigation of cognitive processes in the human brain[3) In general, the brain has phenomenally two heterogeneous properties, that is, one is ‘ material ’ and the other is ‘ non-material'. When we receive visual or auditory information from the external world, conscious experiences of the information (i.e., non-material) occur synchronously with neural activations in the brain cortex (i.e., material). The research method for investigating material phenomena of the brain

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(i.e., neuro-physiological processes) is ' external observation ' as used in all other sciences, whereas for the examination of non-material phenomena (i.e., consciousness), only ' internal observation ' (introspection) is available, though it has long been criticized as a subjective and hence non-scientific method in conscious psychology [4]. In this respect, we can not directly explain mental phenomena but can mediately conjecture them through the corresponding physiological phenomena.

However, if we regard languages as the material of the brain science, we can observe externally the cognitive mental phenomena through the verbal behaviors. If so, it may be possible to explain those internal conscious events by analyzing the verbal processing topologically. In fact, an attempt was already made from this viewpoint to investigate mediately the neuro-mechanism activated dynamically [5] in the living brain, applying topology to the phenomena of verbal thinking [6]. Using topology, Shibahara (2007)[7] recently tried to explain why a set of three words, while each word stores its own meaning respectively in the brain, generate a new continuous meaning as seen in a normal sentence.

The purpose of this paper was to present a brief summary of the topological explanation of a new continuous meaning generated by a set of three words proposed by Shibahara (2007)[7] and then to show further application of topology to verbal thinking.

A Brief Summary of Topological Explanation of Continuity of Meaning Generated by a Set of Three Words Proposed by Shibahara (2007) [7]

When we see an object ' ball ' in the real world (or space), it is transformed to an image (in an image space) and a concept (in a concept space) at the same time. The mental products, an image of ' ball ' and a concept of ' ball ', are further mapped onto a word (i.e., ball) in a word space.

Let's look at a set of words {boy (S), ball (O), throw (V)} and take up the sentence " A boy throws a ball" as a word space X, where the composition of the word space (the set of words) is regarded as a topological space [8]. Then, paying attention to semantic neighborhoods [9] of each subset of X and taking account of whether the subset is semantically closed or open, we perform ' closure ' operation (denoted by ' $-$ ') to each subset. The procedures of closure operations are as follows:

Both words ' boy ' and ' ball ' which exist independently as objects in the real space are semantically closed: $\overline{S} = S$, $\overline{O} = O$, and $\overline{SO} = SO$[10]. On the contrary, the word ' throw ' is a temporary state of either object. Hence, it is semantically not closed but open, and for it to be closed semantically we add SO (the complement of V in X) to V: $\overline{V} = SVO = X$. The two words ' boy ' and ' throw ' are semantically not closed, then, by adding the word ' ball ' to them these three words become semantically closed: $\overline{SV} = SVO = X$. Similarly, $\overline{VO} = SVO = X$. Further, in topology, it is defined that the total space X and the empty set ϕ which is the complement of X are closed: $\overline{X} = X$, $\overline{\phi} = \phi$. In addition, X and ϕ are open by the complementary relation between them. Hence, X and ϕ are respectively closed and open. The results obtained are:

Closures ($\overline{M} = M \cup M^d$: M^d is a set of all limit points of M): $\overline{S} = S$, $\overline{V} = X$, $\overline{O} = O$, $\overline{SV} = X$, $\overline{SO} = SO$, $\overline{VO} = X$, $\overline{X} = X$, $\overline{\phi} = \phi$.

Closed sets ($\overline{M} = M$): S, O, SO, X, ϕ

Open sets (X $-$ closed set(s)): VO, SV, V, ϕ, X

Neighborhoods: $V \subset SV$, $VO \subset SVO = X$

The above data fully satisfy the following **Definition of Connectedness** in the topological space X. " Let X be a topological space. Then, X is called ' connected ' if and only if there is not any closed and open subset except X and ϕ. " Thus, it is clear that the sentence composed of the three words is topologically as well as semantically connected. This implies that in the word space X a new continuous meaning which corresponds to a coherent meaning of the sentence is generated by the three fragmental words.

Next, let's consider the sentence " A boy eats a ball " (word space X) which is syntactically appropriate but semantically anomalous, and we perform closure operation to each subset {boy (S), ball (O), eat (V)} similarly.

Closures: $\overline{S} = S$, $\overline{V} = SV$, $\overline{O} = O$, $\overline{SV} = SV$, $\overline{SO} =$

SO, $\overline{VO} = X$, $\overline{X} = X$, $\overline{\phi} = \phi$

Closed sets: S, O, SV, SO, X, ϕ

Open sets: V, O, SV, VO, ϕ , X

∴Both closed and open subsets in X are: O, SV, X, ϕ .

From the fact that the subsets O and SV are closed and open, it follows that by the definition of connectedness of topological space, the word space X is topologically disconnected. The result suggests that the sentence composed of a set of three words {boy, eat, ball} is semantically disconnected or incoherent.

Topological Explanation of Thinking Processes under the Task of Composing a Simple Sentence of Three Noun Words

From now on, we consider the composing process which works under the task that requires us to compose a simple sentence by using three noun words. For example, when 53 university students were asked to compose a simple sentence by using the three nouns ‘ boy ’, ‘ ball ’, and ‘ river ’, about 75% of them composed a simple sentence, “ A boy dropped a ball into the river, ” with some variations (e.g., The ball thrown by a boy was dropped into the river.). The rests were compound sentences (e.g., A boy was playing with a ball, and the ball was dropped into the river.), or complex sentences (e.g., When a boy threw a ball, the ball was dropped into the river.).

Such a composing process is a kind of thinking called ‘ an inner volitional act ’ by W. Wundt[11)]. From the fact that a thought is spoken by a set of words, it follows topologically that there exists a homeomorphic correspondence[12)] between thought and language. This implies that the thinking process (i.e., composing process) can be topologically conjectured from the set of three words.

Here, let us try to explain topologically the composing process of the most probable sentence, “ A boy dropped a ball into the river. ” Now, let the set of four words be a topological space E, and let ‘ boy ’, ‘ ball ’, ‘ river ’, and ‘ drop ’ be a, b, c, and d, respectively. Then, the following results are obtained.

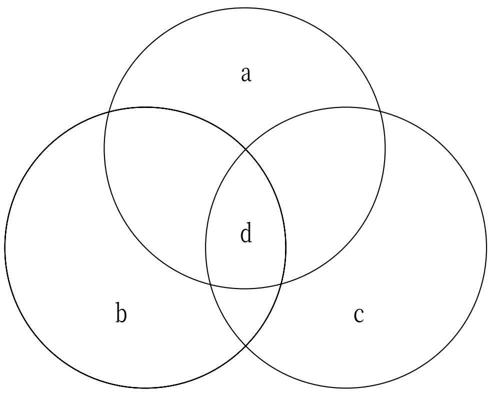
All subsets of $E = 2^4 = 16$: a, b, c, d; ab, ac, ad, bc, bd, cd; abc, abd, acd, bcd; E (abcd) and ϕ (empty set)

Closures: $\overline{a} = a$, $\overline{b} = b$, $\overline{c} = c$, $\overline{d} = E$; $\overline{ab} = ab$, $\overline{bc} = bc$, $\overline{ac} = ac$; $\overline{ad} = E$, $\overline{bd} = E$, $\overline{cd} = E$; $\overline{abc} = abc$, $\overline{abd} = E$, $\overline{bcd} = E$, $\overline{acd} = E$; $\overline{E} = E$, $\overline{\phi} = \phi$

Closed sets: a, b, c, ab, ac, bc, abc, E, ϕ

Open sets: d, ad, bd, cd, abd, acd, bcd, E, ϕ

There is not any closed and open subset except E and ϕ. Hence, the above results indicate that the simple sentence “ A boy dropped a ball into the



a = boy, b = ball, c = river, d = drop: common neighborhood of a, b, and c.

Figure 1. A topological representation by Ben's Figure of the composition, “ A boy dropped a ball to the river, ” on the semantic level

river. " is semantically closed and connected (i.e., coherent).

References and Notes

1. Information Processing Systems of Man: language and brain science. Computer Today (Jap. Science co.). 103, 2001

2. Osaka, N: Consciousness and Working Memory. *Japanese Psychological Review*. 41, 87, 1998

3. Osaka, N: *Cognitive Science of Consciousness: Neural Correlates of Consciousness* (in Japanese), Kyoritsu Shuppan, Tokyo, 2000

4. Köhler, W: *Gestalt Psychology*, Liveright, New York, 1947

5. The term ' dynamic ' in this context implies a psychological level of dynamics based on the phenomena observed in the experiment of conditioned reflex (M. J. Bass and C. L. Hull, The Irradiation of a Tactile Conditioned Reflex in Man. *Journal of Comparative Psychology*, 17, 1934) and on W. Köhler's dynamic view (*Dynamics in Psychology*. Liveright, New York, 1940)

6. Topology is one area of modern mathematics of relatively recent origin, and it bears most abstract, qualitative, and logical character. One who first applied topology to psychology was K. Lewin, a Gestalt psychologist. He published a book entitled ' Principles of Topological Psychology ' (McGraw-Hill, New York, 1936). However, his application of topology was insufficient, because topology which treats problems of infinity and continuity is the most abstract and abstruse mathematics, and probably because topology in his days was not so progressed. Then, S. Shibahara attempted to apply topology perfectly to psychology in 1972 (a doctoral thesis at Kyoto University ' Topological Approach to Thinking Psychology '), and further developed this theoretical system in 1987 (see ' Mathematical Principles of Mental Philosophy ' , Tottori Press, Japan).

7. Shibahara, N.: Topological explanation of continuity of meaning generated by a set of words in the brain. *Journal of Kinki Welfare University*, 8, 87-90, 2007

8. A topological space X is a family or system of subsets of X which satisfies any one of the following topological axioms: closures, open kernels, closed sets, open sets, and neighborhoods. For example, the axiom of a family of closed sets in X: a Any intersection of closed sets in X is closed, s Any finite union of closed sets in X is closed, and d The empty set ϕ and the total space X are both closed. From this axiom of closed sets, the axiom of open sets is deduced by the duality of closed sets and open sets: a Any union of open sets in X is open, s Any finite intersection of open sets in X is open, and d The empty set ϕ and the total space X are both open. In addition, a closure operation leads to a closed set in such a way: $\overline{M} \Rightarrow M =$ a closed set. Similarly, an open kernel operation leads to an open set: $M^i \Rightarrow M =$ an open set.

9. In topology, the term ' neighborhood '(nbd) is defined as the interior of a circle or ball such that the center is a point p and the radius is a positive real number ε. In this context, however, the ' semantic nbd ' of a subset x in a word space X implies all that are associable or associated with the x, on an image space, a concept space (both image and concept spaces are semantic spaces of the word space), or the word space.

10. $\overline{S} \cup \overline{O} = \overline{SO}$, that is, the union of two closed sets is a closed set (cf. the second item of Axiom of closed sets in note 8).

11. Wundt, W.: Grundriss der Psychologie. Leipzig. 1920

12. The topological term ' homeomorphism ' is defined as follows:" Let X and Y be topological spaces. A map f: X → Y is a homeomorphism if and only if f is one-to-one, onto and f and $f-1$ are both continuous "(Let V be the nbd of a in X and U be the nbd of b in Y, the map f: $V_a \rightarrow U_b$ and the inverse map f^{-1}: $U_b \rightarrow V_a$ are called" both continuity ").