〈研究ノート〉

近畿福祉大学紀要
J. Kinki Welf
Vol. 8 (1) 87〜90 (2007)

単語の集合による新たな意味生成の位相数学的説明

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Topological Explanation of Continuity of Meaning Generated by a Set of Words in the Brain

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チョムスキーは、話者がどのようにして無限の文法的に適った文を表出し、理解するのかを説明する生成文法理論を構築した。以来、言語と思考に関する認知科学的研究が盛んに行われるようになったが、コンピューターに代表される情報科学の長足の進歩と脳科学の新たな展開がこれに拍車をかけた。本研究において、チョムスキーの文法、つまり意味ではなく統語法を優先する文法構築に対する一つの挑戦として、何故、我々は３つの単語の集合から文としての新しい意味を生成するのかを、位相数学（トポロジー）の応用によって説明することを目的とした。

Key Words： Topology, Word, Meaning, Set, Mapping
トポロジー、単語、意味、集合、写像

From the view of cognitive science on thought and language, the following question occurs to our mind. " Why can a set of words, while each word stores its own meaning in the brain respectively, generate a new continuous meaning as seen in a normal sentence ? " If each word has its own meaning independently, the sentence composed of these words will be semantically discontinuous, although in reality this will not happen except for the schizophrenic thought [1] and the like. However, this issue has been treated as a self-evident fact in syntax by many linguists [2]. Therefore, I attempted to resolve the issue semantically by using topological spaces [3-6] as mentioned below.

Since the problem of generation of a new continuous meaning by a set of words is deeply relevant to an image space, a conceptual space, and a word space, these mental spaces together with the mappings between them are elucidated here (see Fig. 1).

Four kinds of spaces (one real and three mental) and two kinds of mappings (primary and secondary) among them illustrate full information processing in cognition [7]. A " Real Space " is an actually existing objective space, that is, the world wherein we live and experience various things. Through the sense organs, we can map any objects existing in the real space into the brain cortex. The process is called ' perception ' and the mapped image is called a ' percept ', and a set or a family of images is called an " Image Space (concrete space) ". By the mental functions of generalization and abstraction, a set of similar images is transformed or mapped into a concept. A set or a family of concepts in the brain cortex is called a " Concept Space (abstract space) ". The mental products, such as images in the image space and concepts in the concept space, are fur-

受付　平成19年５月７日，受理　平成19年６月９日
近畿福祉大学　〒679-2217　兵庫県神崎郡福崎町高岡 1966-5

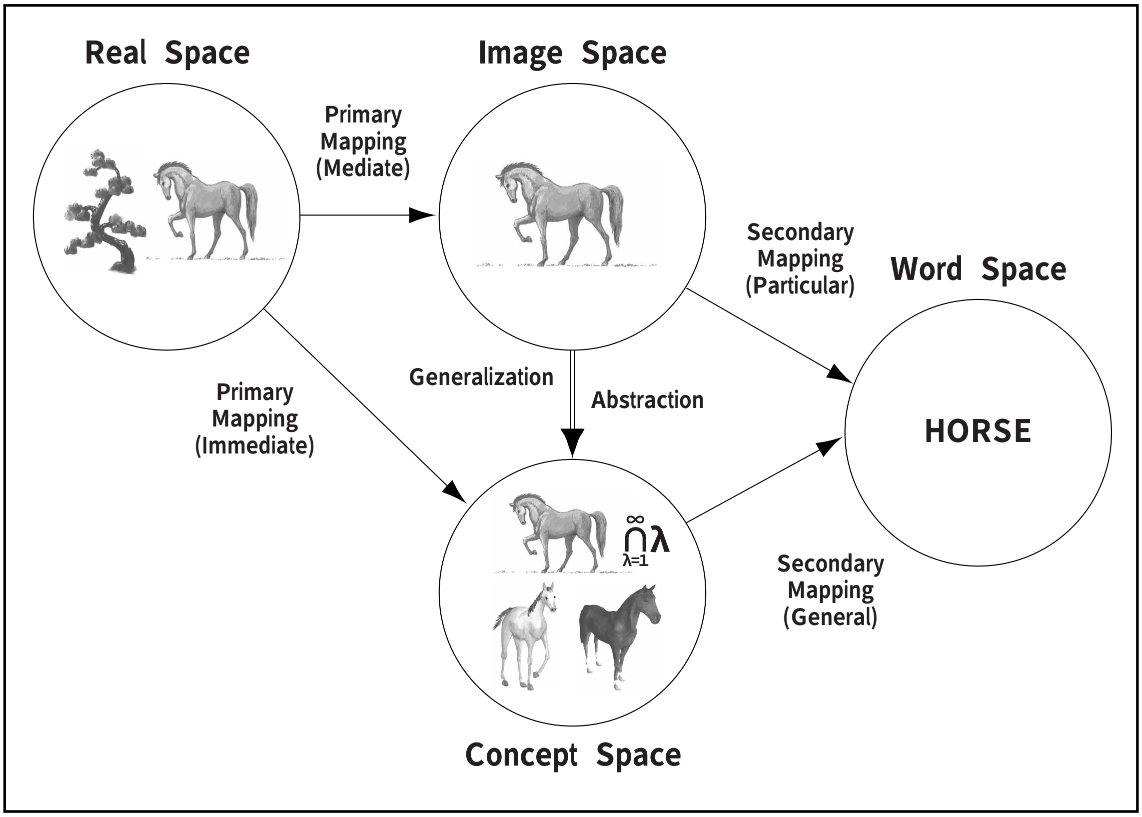

Figure 1. Four kinds of spaces and two kinds of mappings

ther mapped onto words or signs in a "Word Space", with the result that not only can we utilize words at will on the space, but also via words freely communicate with one another (sign space).

From the above mentioned, it follows that both image and concept spaces are semantic spaces of the word space, and reversely the latter is a sign space of the former two spaces. In addition, for completion of the word space, we postulate that everything which is the object of our cognition and thinking always can be verbalized, i.e., mapped into the word space.

Application of Topological Concepts to Generation of New Continuous Meaning in Word Space

In order to explain topologically the generation of a new continuous meaning by a set of words, we take up the sentence L_1 "A boy throws a ball" as Word Space X_1 (see Fig. 2).

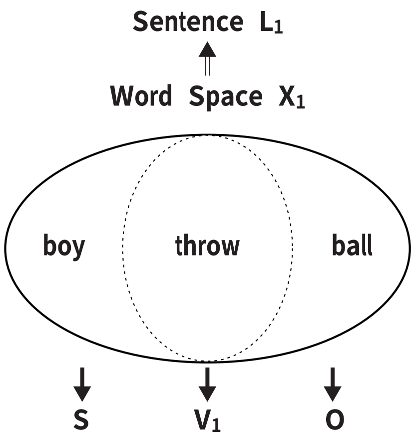

Figure 2. Word space X_1 composed of a set of three words

As shown in Fig. 2, Word Space X_1 is composed of a set of three words, 'boy', 'throw', and 'ball'. Each word which has an independent meaning is arranged in a syntactical order, i.e., Subject, Verb, and Object, in the word space X_1. Accordingly, all subsets of the total space X_1 which consists of three elements (words) are: {S, V, O, SV, VO, SO, X, ϕ} $= 2^3 = 8$ subsets.

Now, paying attention to semantic neighbourhoods of each subset of X_1 and taking account of whether the subset is semantically closed or open, we perform 'closure' operation to each subset. The procedures of closure operations are as follows:

The word 'boy' or 'ball' is a thing (entity) which exists independently in the real space. Hence, either is semantically closed: $\overline{S} = S$, $\overline{O} = O$, and $\overline{SO} = SO$. On the contrary, the word 'throw' is a temporary state of either thing. Hence, it is semantically not closed but open, and for it to be closed semantically we add SO (the complement of V in X) to V: $\overline{V} = SVO = X$. The two words 'boy' and 'throw' are not semantically closed, then, by adding the word 'ball' to them these three words become semantically closed: $\overline{SV} = SVO = X$. Similarly, $\overline{VO} = SVO = X$. Moreover, in topology, it is defined that the total space X and its complement ϕ are closed (and open): $\overline{X} = X$, $\overline{\phi} = \phi$. The results obtained are:

Closures ($\overline{M} = M \cup M^d$ or $M \cup$ a set of all limit points of M): $\overline{S} = S$, $\overline{V} = X$, $\overline{O} = O$, $\overline{SV} = X$, $\overline{SO} = SO$, $\overline{VO} = X$, $\overline{X} = X$, $\overline{\phi} = \phi$.

Closed sets ($\overline{M} = M$): S, O, SO, X, ϕ

Open sets (X − closed set(s)): VO, SV, V, ϕ, X

The above data fully satisfy the following "Definition of Connectedness"[3] in the topological space X. "Let X be a topological space. Then, X is called 'connected' if and only if there is not any closed and open subset except X and ϕ." Thus, it is clear that the sentence L_1 composed of the three words is topologically as well as semantically connected. This implies that in Word Space X_1 a new continuous meaning which corresponds to a coherent meaning of the sentence L_1 is generated by the three fragmental words. Thus, we could answer the question raised earlier.

A Kind of Counter Example of the Above Illustration

We now consider the sentence L_2 "A boy eats a ball" which has the same syntactic structure S-V-O as the sentence L_1 (see Fig. 2), but the verb in the sentence L_2 is semantically different from that in the sentence L_1, with the result that the sentence L_2 is syntactically appropriate but semantically anomalous (see Fig. 3).

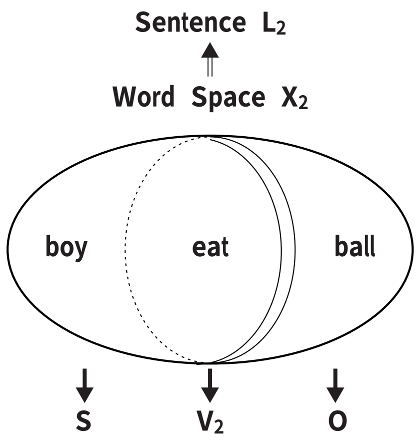

Figure 3. Word space X_2 composed of a set of three words

As shown in Fig. 3, Word Space X_2 is composed of a set of three words, 'boy', 'eat', and 'ball'. This is different from Fig. 2 only in the verb, i.e., V_1 (throw) $\Rightarrow$ V_2 (eat), with the result that $V_2 \cap O = \phi$. Hence, similar to the case of the word space X_1 in Fig. 2, all subsets of the word space X_2 are: {S, V, O, SV, VO, SO, X, ϕ} $= 2^3 = 8$. Now, we perform closure operation to each subset of X_2.

Then, **Closure:** $\overline{S} = S$, $\overline{V} = SV$, $\overline{O} = O$, $\overline{SV} = SV$, $\overline{SO} = SO$, $\overline{VO} = X$, $\overline{X} = X$, $\overline{\phi} = \phi$.

Closed sets: S, O, SV, SO, X, ϕ.

Open sets: V, O, SV, VO, ϕ, X

$\therefore$ Both closed and open subsets in X_2 are: O, SV, X, ϕ.

From the fact that the subsets O and SV are closed and open, it follows that by the definition of connectedness of topological space, the word space X_2 is topologically disconnected. The result suggests that the sentence L_2 composed of a set of three words {boy, eat, ball} is semantically disconnected or incoherent.

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